Lecture 7: Indices and Logarithms

7.1 Introduction

We have already learned how to solve several different types of equations – linear equations, quadratic equations, etc. In this section, we learn how to solve two new types of equations – exponential and logarithmic equations.

In mathematics sometimes students find difficult or cumbersome in computing when dealing with exponents or decimal numbers. To overcome this problem, we normally need the use of rules of exponents or a logarithm table or a calculator with logarithm function. Therefore this lecture is designed to introduce you to the concepts and theories of Indices and Logarithms and their applications in calculating arithmetical and algebraic problems. To assist you in using logarithm in computing, copies of four figure tables of common logarithms, antilogarithms and natural logarithms have been given at the end as an appendix.

Learning Objectives

At the end of the lecture, you should be able to:

i. Define logarithm;
ii. Identify the properties of logarithms;
iii. Explain common logarithms and change of base;
iv. Determine common antilogarithms;
v. Discuss natural base and natural logarithm.

When multiplying powers of the same quantity or unit add up the exponents as shown below:

$x^a x^b = \quad x^{a+b}$ ...(1)

Example:

$4^2 . 4^3 = 4^5 = 1024$

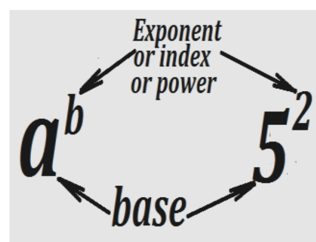
7.2 Idea of negative and rational indices

There are types of indices or exponents and each type has a specific interpretation. the first type are the rational exponents which combine powers and roots of the base, while on the other hand the second one are the negative exponents which indicate that the reciprocal of the base is to be applied.

Exponents

The exponent of a number can be defined as how many times the same number is use in a multiplication process.

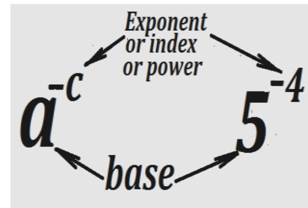
The Multiplication Process.	Explanation
$5 \times 5 = 5^2$	5^2 is five to the second power
$5 \times 5 \times 5 = 5^3$	5^3 is five to the third power
$5 \times 5 \times 5 \times 5 = 5^4$	5^4 is five to the fourth power
∘	∘
∘	∘
∘	∘
$5 \times 5 \times 5 \ldots \times 5 \; to \; n \; times = 5^n$	5^n is five to the nth power



Negative Exponents

The negative exponent of a number can be defined as how many times the same number is divided in a division process.

The Division Process.	Explanation
$1 \div 5 \div 5 = 1 \div (5 \times 5) = 5^{-2}$	5^2 is five to the second power
$1 \div (5 \times 5 \times 5) = 5^{-3}$	5^3 is five to the third power
$1 \div (5 \times 5 \times 5 \times 5) = 5^{-4}$	5^4 is five to the fourth power
∘	∘
∘	∘
∘	∘
$1 \div (5 \times 5 \ldots \times 5 \ to\ n\ times) = 5^{-n}$	5^n is five to the nth power



Examples of exponent equations are:

$$2^x = 8; \qquad 3^{y+1} = 10; \text{ and } 5^{a^2} = 5^{2a-1}$$

We can solve some exponential equations using one-to-one property of the exponential function.

Suppose $a > 0$ and $x \neq 1$.

If $a^{x_1} = a^{x_2}$, then $x_1 = x_2$

Example:

Solve $5^x = 25$

Solution:

Write both sides with same base 5

$5^x = 5^2$

7.3 Logarithm to any Base

What are Logarithms?

The exponential function $y = 2^x$ is a one-to-one function, since its graph passes the horizontal line test. This implies that its inverse $x = 2^y$ is a function. However, we currently have no way of solving $x = 2^y$ for y. To solve $x = 2^y$ for y, we introduce the following definition.

Suppose $a > 0$ and $a \neq 1$, then the logarithmic function with base a is function of the form

$y = \log_a x$, where

$y = \log_a x$ means $x = a^y$

Exponent Form	Logarithmic Form
$x = a^y$	$y = log_a x$

a is a base for both exponent and logarithmic function.

This implies that, $log_a x$ is the exponent to which a must be raised in order to obtain x.

Take Note

Definition of $\log_a x$

1. When we write $y = \log_a x$ to mean $x = a^y$, where $a > 0$ and $a \neq 1$.

   $y = \log_a x$ is read y is the logarithm of x to the base a

   The definition of the logarithmic function allows us to write the equation $x = a^y$ in two ways – in exponential form or in logarithmic form

1. A logarithm is just an exponent.

Theorem of Logarithmic Functions

(ii) $\log_a a = 1$

(iii) $\log_a 1 = 0$

Note that,

$a^1 = a$ and $a^0 = 1$ respectively

Example:

$\log_2 8 = 3$ since $2^3 = 8$

$\log_5 \frac{1}{25} = -2$ since $5^2 = \frac{1}{25}$

$\log_{10} 10,000 = 4$ since $10^4 = 10,000$

Example:

Solve the equation: $\log_4 (5 + x) = 3$

Solution:

If $\log_4 (5 + x) = 3$, then by the definition of logarithm,

$$5 + x = 4^3 \text{ or } 5 + x = 64$$

$$x = 64 - 5 = 59$$

$$x = 59$$

Applying the one-to-one property of the exponential function to equate exponents, then we have $X = 2$.

Example:

Solve $5^x = 7$

Solution:

$$5^x = 7$$

take the log of each side, we have

$$\log 5^x = \log 7$$

$$x\log 5 = \log 7$$

$$x = \frac{\log 7}{\log 5} » \frac{0.8451}{0.6990} » 1.21$$

7.4 Logarithms to Solve Equations

Logarithmic Equations

A logarithmic equation is an equation that contains a logarithm of a variable quantity.

Examples of logarithmic equations are:

(i) $\log_2 x = 5$

(ii) $\log_6 x\,(x + 1) = \log_6 7$ and

(iii) $\log y - \log 4 = 3$

We can solve some logarithmic equations using one-to-one property of the logarithmic function as shown below.

Suppose a, x_1, and x_2 are positive numbers and $a \neq 1$.

If $\log_a x_1 = \log_a x_2$, then

$$x_1 = x_2$$

Example:

Solve $\log_7 (2x + 5) = \log_7 11$

Solution:

Both logarithms have the same base. Therefore,

$$2x + 5 = 11$$

$$2x = 11 - 5 = 6$$

$$x = \frac{6}{2} = 3$$

$$x = 3$$

Example:

Solve: $\log (x + 1) - \log (x - 2) = 1$

Solution:

$\log \frac{x+1}{x-2} = 1 \Rightarrow \frac{x+1}{x-2} = 10^1 \rightarrow$ (exponential form)

That is,

$\frac{x+1}{x-2} = 10$

$x + 1 = 10x - 20$

$x = \frac{21}{9} = \frac{7}{3}$

The solution set = $\frac{7}{3}$

Properties of Logarithms

Historically, logarithms were developed to simplify complex numerical computations. The availability of calculators and computers has made this use of logarithms obsolete. Nevertheless, the properties of logarithms, still serve as the basis for using logarithms for both numerical and non-numerical purposes.

The following are common properties of logarithm:

Product Rule

This is also known multiplication rule. The product rule states that the log of a product is equal to the sum of the logs.

Let $u = b^m$ and $v = b^n$ and let's write these exponential statements in logarithmic:

$u = b^m$ is equivalent to $\log_b u = m$

$v = b^n$ is equivalent to $\log_b v = n$

Using product rule, we have,

$u.v = b^m \cdot b^n = b^{m+n}$ which is equivalent to

$\log_b (uv) = m + n$ ...(2)

If we substitute m = $\log_b u$ and n= $\log_b v$ into equation (2), we get;

$\log_b (uv) = \log_b u + \log_b v$ ...(3)

This is exactly what the product rule states.

Take Note

The logarithm of a product rule equals the sum of the logarithms of its factors. That

$\log_b (uv) = \log_b u + \log_b v$

When the base is the same, we also write

$\log (uv) = \log u + \log v$

Example:

Express the following as a sum of simpler logarithm:

$\log_b (x^3 y)$

Solution:

$\log_b (x^3 y) \quad = \log_b x^3 + \log_b y$

Quotient Rule

This also is known Division Rule. The quotient rule is expressed as:

$\log_b \frac{u}{v} = \log_b u - \log_b v$

When dividing the same base, we normally subtract the exponent of the denominator from the exponent of the numerator.

That is: $\frac{b^x}{b^y} = b^{x-y}$

Take Note

The logarithm of a quotient equals the logarithm of the nominator minus the logarithm of the denominator. That is,

$$\log_b \frac{u}{v} = \log_b u - \log_b v$$

Example:

1. $\frac{3^5}{3^2} = 3^{5-2} = 3^3 = 27$

2. $\log_b \frac{\sqrt{x}}{r^5} = \log_b \frac{x^{\frac{1}{2}}}{r^5}$

$= \log_b x^{\frac{1}{2}} - \log_b r^5$

Power Rule

The power rule says that the log of a power is the exponent times the log. The power rule is given by:

$$\log_b x^y = y\log_b x$$

Also written as

$$\log_b \sqrt{x} = y\log_b x$$

Take Note

The logarithm of the y^{th} power of a number equals y times the logarithm of the number. That is,

$\log_b x^y = y\log_b x$

Also $\log_b$ $\log_b \sqrt[y]{x} = \frac{1}{y}\log_b x$ holds water.

That is, the logarithm of the real positive y^{th} root of a number equals $\frac{1}{y}$ times the logarithm of the number

Example:

(i) $\log_{10} x^5 = 5\log_{10} x$

(ii) $\log_a 48$ rewrite 48 as a product factors of 2 and 3

$48 = 2^4.3$

$\log_a 48 = 2^4.3 = \log_a 2^4 + \log_a 3$

$= 4\log_a 2 + \log_a 3$

$= 4(1.2) + 1.38$

$= \underline{6.18.}$

Common Logarithms

Before electronic calculators were inverted, logarithms with base 10 were used for complicated numerical computations involving products, quotients, and powers of real numbers. Base 10 was employed because it is well suited for numbers that are expressed in decimal form and are more efficient in many situations than logarithms with other bases. Logarithms with base 10 are called common logarithms. For simplicity, $\log_{10} x$ is written log x. That is;

$\log x = \log_{10} x$

Take Note

Definition of Common Logarithms
log x = log x for every x > 0

Since inexpensive calculators are now available, there is little need for common logarithms as a tool for computational work. However, base 10 does occur in applications and therefore many calculators have a log key that can be used to approximate common logarithms. Appendix 2 contains a table of common logarithms that may be used if a calculator is not available or is inoperative.

We can determine common logarithms of powers of 10 using the rule $\log_a a^r = r$ where r is the logarithm of a to base a. likewise we have,

$\log_{10} 100 = \log_{10} 10^2 = 2\log_{10}10 = 2$

$\log_{10} 1000 = \log_{10}10^3 = 3 \log_{10} 10 = 3$

$\log_{10} 10000 = \log_{10} 10^4 = 4 \log_{10} 10 = 4$

$\log_{10}10 = \log_{10} 10^1 = 1 \log_{10} 10 = 1$

$\log_{10} 10 = \log_{10}10^0 = 1$

$\log_{10} 0.1 = \log_{10}10^{-1} = -1$

$\log_{10} 0.01 = \log_{10} 10^{-2} = -2$

$\log_{10} 0.001 = \log_{10} 10^{-3} = -3$

Note that numbers greater than 1 have common logarithms that are positive and numbers between 0 and 1 have common logarithms that are negative.

To determine common logarithms of other positive numbers, we use either a table or a calculator. Appendix 1 of this unit contains a table of common logarithms.

Since a calculator is faster than a table, we will use a calculator. However a logarithm table will be used in some cases to enable you understand its usefulness in computation.

Example:

Find log 55

Solution:

Press the following keys on your calculator:

Clear	55	log	1.740362689

Rounding off to four decimal places, we have log 55 $\approx$ 1.7404

You can use your calculator to check the answer to example given above by showing that the equivalent exponential equation below is true.

That is,

$10^{1.7404}$ » 55

To do this, press

Clear	10	y^x	1.7404	=	55.00472529

If a number is too large or too small to be entered into your calculator in standard form, you must first write the number in scientific notation.

The following examples illustrate this situation.

Example:

Find each common logarithm to four decimal places.

(a) log 437,000,000,000

(b) log 0.00000000437

Solution:

(a) log 437,000,000,000 = log (4.37×10^{11}) Þ scientific notation

log 4.37 + log 10^{11} Þ product rule for logarithm

» 0.6405 + 11

$\approx$ 11.6405

(b) log 0.00000000437 = log (4.37×10^{-9})

$= \log 4.37 + \log 10^{-9}$

$\approx 0.6405 + (-9)$

≈ -8.3595

Standard Notation

In the example above, we have pointed out the use of standard notation for a number which is too large or too small to be entered into your calculator. In order to obtain a logarithmic solution for these numbers, we need to express them in scientific notation together with a table of common logarithms.

For example, to find log 495 we follow the following procedure:

log 495

Write 495 in scientific notation, then

$\log 495 = \log (4.95 \times 10^2)$ (Use the product rule for logarithms)

$= \log 4.95 + \log 10^2$ (By power rule)

$= \log 4.95 + 2 \log 10$ since $2\log 10 = 2$

then we get,

$= \log 4.95 + 2$

In order to find log 4.95 we look at the table of common logarithms given in Appendix 1.

We look down the first column labeled N to the row containing 4.9, and them go across to the column with 5 at the top (or bottom). The entry we get in row 4.9 and column 5 is.6946.

Therefore, we have,

$\log 495 = \log 4.95 + 2$

$= 0.6946 + 2$

$= 2.6946$

The decimal part of the logarithms (.6946) is called the mantissa while the integer part of the logarithm (2) is called the characteristic.

Note that the mantissa which is obtained from the table is only a four-decimal approximation. When a calculator is used we obtain the seventh decimal approximation 2.6946052. Therefore, the log 495 = 2.6946.

Also note that the mantissa is always positive.

Example:

Find the following logarithms.

1. log 28,300

2. log .0749

Solution:

1. $\log 28{,}300 = \log (2.83 \times 10^4)$

$= \log 2.83 + \log 10^4$

$= \log 2.83 + 4 \log 10$

$= \log 2.83 + 4$ the characteristic is 4; look log 2.83 in the table you will get 0.4518

$= 0.4518 + 4$

$= 4.4518$

2. $\log .0749 = \log (7.49 \times 10^{-2})$

$= \log 7.49 + \log 10^{-2}$

$= \log 7.49 + (-2) \log 10$

$= \log 7.49 - 2 \log 10$ log 7.49 =. 8745 from the table and log 10 =1

$= 0.8745 - 2(1)$

$= 2.8745$

From the above two examples, we can generalize that the characteristic of a common logarithm is simply the exponent of 10 when the number is scientific notation.

For more understanding on the scientific notation, logarithms notation and logarithm, let us consider the following narration below:

Number	Scientific Notation	Logarithmic Notation	Logarithm
10,000	1.0×10^4	$\log_{10} 10000$	4.0000
1,000	1.0×10^3	$\log_{10} 1000$	3.000
10	1.0×10^1	$\log_{10} 10$	1.000
0.1	1.0×10^{-1}	$\log_{10} 0.1$	1.000
0.01	1.0×10^{-2}	$\log_{10} 0.01$	2.000

Expressing numbers in a scientific notation assists in identification of the characteristics.

Activity

Explain with examples how can you differentiate between characteristics and mantissa.

Common Antilogarithms

Sometimes we want to reverse the procedure of finding the logarithm of a number. For example, if we have $\log 100 = 2$, we say that 2 is the logarithm of 100. on the other hand, we say that 100 is the antilogarithm (denoted: antilog of 2).

Take Note

Definition of Antilogarithms

If $\log x = y$ then $x = \text{antilog } y$

Example:

Find x, if $\log x = 2.49$

Solution:

There are two ways to do this on most calculators.

Method 1

Use the inverse of the log function

Clear	2.49	INV	log	309.0295433

Therefore, $x = \text{antilog } 2.49 \approx 309$ (three significant figures)

Method 2

Use the fact that $x = 10^{2.49}$

Clear	10	y^x	2.49	=	309.0295433

The answer to three significant figure is $x = \text{antilog } 2.49$ » 309

We can verify the solution to the above example by substituting it into the original equation. That is,

$\log 309 \approx 2.49$

Example:

Find x to three significant figures if log x = 37.98

Solution:

Press:	Clear	37.98	INV	log	9.5499258637

The antilog a rhythm of 37.98 is very large, so your calculator expressed the answer in Scientific Notation. That is,

x = antilog 37.98 $\approx 9.55 \times 10^{37}$

The two methods used above to determine the antilog of a number need a use of calculator. However, we can also use a table of antilogarithms to obtain the solution needed.

In the appendix there are also tables for antilogarithms. These are numbers whose logarithms are given.

Example:

Given,

log 28,300 = 4.4518,

then, antilog 4.4518 = 28,300

Example:

Find x if log x = 3.9299

Solution:

We look at log$x$ = 3.9299 as log$x$ = .9299 +3, we can note that the characteristic is 3. If we look in the table of antilogarithm, we find the mantissa .9299 is located as follow:

At intersection of the row .92 and column 9 = 8492

Add the value at intersection of .92 and 9 (the 4^{th} digit in the mantissa) = 17

That is, 8492 + 17 = 8509

log x =3.9299

x =.9299 + 3, 3 is the characteristic, .9299 is the mantisa (the number of mantisa is read in the antilogarithm table)

$x = 8.509 \times 10^3$

$x = 8509$

The process shown in the above example is called finding the antilogarithm of 3.9299. The example above could also be worked as find antilong 3.9299.

Example:

Find antilog (6.5391 – 10)

Solution

If you are asked to find N given that

$\log N = 6.5391 - 10$

$\log N = 6.5391 - 10$, The characteristic is 6 – 10= – 4 the mantissa is .5391.

$\log N = .5391 - 4$

We find .5391 in the antilogarithm table as follows:

At intersection of the row .53 and column 9 = 3459, then add the value at the intersection of .53 and 1 (the 4th digit in the mantissa) = 1

That is 3459 + 1 = 3460

Therefore,

$$\log N = 6.5391 - 10$$
$$\log N = .5391 - 4$$
$$N = 3.46 \times 10^{-4}$$
$$N = .000346$$

Example:

Find antilog 1.426

Solution:

log N= 1.426 1 is the characteristic .426 is the mantissa

Find the number using the mantissa .426 by reading the intersection of 42 row and 6 column (in the antilogarithm table), we get 2667.

Therefore, the number for which logarithm is 1.426 (antilog 1.426) is 2.667×10^1

Example:

Find antilog 3.8763

Solution:

log x=3.8763

x=.8763 + 3 3 is the characteristic, while .8763 is the mantissa

Finding the number in the antilogarithm table using the mantissa .8763 by reading the intersection of .87 row and 6 column, we get 751, and then add the mean difference for 3 which is 5, that is

antilog 3.876	=	7516
add mean difference for 3	=	5
		7521

Therefore, the number for which logarithm is 3.8763 is $7.521 \times 10^3 = 7521.0$

Change of Base

Since there an infinitive number of possible bases of or the logarithmic function, it would be impossible to construct a table or calculator key for every base. However, we can find logarithms to any base using the change-of-base formula given below:

Take Note

Change-of-base Formula

The logarithm of a number y, to the base b is equal to the logarithm of the number to the base a divided by the logarithm of b to the base of a.

Let $x = \log_b y$

Then, $b^x = y$ exponential form

$\log_a b^x = \log_a y$ take $\log_a$ of each side

$x\log_a b = \log_a y$ power rule for logarithms

$x = \frac{\log_a y}{\log_a b}$ or $\log_b y = \frac{\log_a y}{\log_b b}$

We can now find logarithms to any base by using the change-of-base formula to convert the logarithm to a common logarithm or to a natural logarithm.

Example:

Convert $\log_3 x$ into a logarithm using base 9.

Solution:

$\log_3 x = \log_9 x / \log_9 3$ since $\log_9 3 = ½$, then

$= (\log_9 x)/ 1/2$

$= 2 \log_9 x$

Example:

Find $\log_2 5$ to four decimal places.

Solution:

Use the change-of-base formula with $b = 2$, x =5 and a =10

$$\log_2 5 = \frac{\log_{10} 5}{\log_{10} 2} » \frac{0.69897}{0.30103} » 2.3219$$

You could also do the above example by converting to natural logarithms instead of to common logarithms as indicated below:

$$\log_2 5 = \frac{\ln 5}{\ln 2} » \frac{1.60944}{0.69315} \approx 2.3219$$

Natural Logarithm

There is a particular number, designated by the letter ℓ , which is called the natural base. The value of natural base (ℓ) is 2.7182818 (i.e. = 2.7182818). The logarithm function with base ℓ is called the natural logarithmic function. We use ln x as an abbreviation for log $_\ell$ x and refer to it as the natural logarithm of x.

Take Note

> Definition of Natural Logarithms
>
> ln $x = \log_\ell x$ for every $x > 0$
>
> The value of log$_e$ (or ln x) = 0.4343.

Laws of Natural Logarithms

(i) $\ln(uv) = \ln u + \ln v$

(ii) $\ln \frac{u}{v} = \ln u - \ln v$

(iii) $\ln u^{\ell} = \ell \ln u$

As we see in common logarithms, many calculators have a key labelled ln x that can be used to approximate natural logarithms. A table of values of ln x is given in on table 3 of Appendix 2.

Example:

Use a calculator to approximate log 436; ln 436; and ln 0.0436

Solution:

log 436 » 2.6394865

ln 436 » 6.0776422

log 0.0436 » –1.3605135

ln 0.0436 » –3.1326981

Example:

Find ln 1350 to four decimal places.

Solution:

Press [Clear] 1350 [ln *x*] [7.2078559871]

Therefore, ln 1350 ≈ 7.2079

To solve certain problems, it is necessary to find x when given either log *x* or ln *x*. One way to accomplish this is by using the inverse function key [INV] If we first press INV and then press log, we obtain the inverse logarithm function $\log^{-1}$.

The $\log^{-1}$ is the exponential function with base 10. Since $\log 10^{-1}$ (log *x*) = *x*, we can obtain *x* by entering log x and then pressing successfully, INV and log. Similarly, given ln *x*, we can find by entering ln *x* and pressing INV and ln *x*. This procedure is illustrated below.

Example:

Find *x* to three significant figures if ln *x* = 3.18

Solution:

[Clear] 3.18 [INV] [ln *x*] [24.04675355]

Therefore, *x* » 24.0

Example:

Find the natural logarithms of the following functions without using a calculator.

(a) $\ln \ell^5$

(b) $\ln \frac{1}{\sqrt{\ell}}$

(b) $\ln\frac{1}{\sqrt{\ell}} = \ln\frac{1}{\ell^{\frac{1}{2}}} = \ln \ell^{-\frac{1}{2}} = -\frac{1}{2}$

Example:

Approximate x to three decimal places if ln $x = 4.7$

Solution:

Given ln $x = 4.7$

Enter: 4.7, then

Press:	INV	In x	:	109.94717

Hence, x » 109.947

7.5 Logarithmic graphs

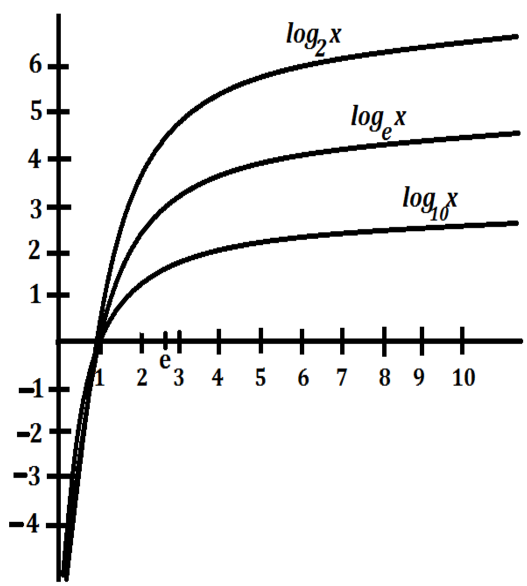
In order to graph a logarithm ($y = log\ x$), substitute one variable (x) in numbers to get values for the other (y), then plot them on a graph and connect the dots.

Remember

The logarithm will never touch or cross the y-axis but will come as close as possible

The logarithm is undefined at zero

The logarithm is undefined at a number less than zero,



The following table lists the three commonly used logarithms

Name for Logarithm	Base (b)	Notation(s) ($log_b x$)	Used in
Binary logarithm; logarithm base two or Log base 2	2	$log_2 x$	Mathematics and information & communication Technology
Natural logarithm; logarithm base e or Log base e (where $e = 2.718$)	e	$log_e x$ or ln	Mathematics, physics, chemistry, statistics, economics, information & communication Technology, and engineering
Common logarithm; logarithm base ten or Log base 10	10	$log_{10} x$ or $logx$	Mathematics, physics, chemistry, statistics, economics, information & communication Technology, and engineering

Summary

Logarithms were developed to simplify complex numerical computations. The product rule states that the log of a product is equal to the sum of the logs. The logarithm of a product rule equals the sum of the logarithms of its factors. The logarithm of a quotient equals the logarithm of the nominator minus the logarithm of the denominator. The logarithm of the y^{th} power of a number equals y times the logarithm of the number. The logarithm of a number y, to the base b is equal to the logarithm of the number to the base a divided by the logarithm of b to the base of a. An exponential equation is an equation that contains a variable in an exponent. A logarithmic equation is an equation that contains a logarithm of a variable quantity.

Exercise

1. Find x if:

(i) $\log_4 2 = x$ (ii) $\log_5 x = 2$

(iii) $\log_x 8 = 3$

2. Write the following equations into logarithmic form

(i) $2^6 = 64$ (ii) $10^2 = 100$

(iii) $3^4 = 81$ (iv) $4^{-2} = \frac{1}{16}$

(v) $4^5 = 1{,}024$ (vi) $16^{1/4} = 2$

(vii) $10^4 = 1000$ (viii) $10^{-2} = \frac{1}{36}$

3. Write the following logarithmic expression in exponential form

(i) $\log_4 64 = 3$ (ii) $\log_{10} 100{,}000 = 5$

(iii) $\log_{10} 0.001 = -3$ (iv) $\log_{1/3} 27 = -3$

(v) $\log_8 \frac{1}{2} = -\frac{1}{3}$ (vi) $\log_k b = m$

(vii) $\log_{16} 2 = \frac{1}{4}$ (viii) $\log_{81} 9 = \frac{1}{2}$

(ix) $\log_8 8 = 1$ (x) $\log_{11} 11 = 1$

4. Evaluate of the following logarithms

(i) $\log_2 8$ (ii) $\log_7 7$

(iii) $\log_3 1$

5. Find the following logarithms:

(a) log 564 (b) log 8.44

(c) log 33592 (d) $\log (7.56 \times 10^{-4})$

(e) log 8005 (f) log .00371

6. Find the following antilogarithms:

(a) antilog 3.6253 (b) antilog 3.22

(c) antilog (8.9766 – 10) (d) antilog 0.1994

(e) antilog (0.8506 – 1)

7. Use the following logarithms to express the given logarithm as a sum of simpler ones.

(i) $\log_5 x^5$ (ii) $\log_4 xyz$ (iii) $\log 8^{\frac{4}{7}}$

(iv) $\log_{10} 10^{-\frac{1}{4}}$ (v) $\log_x x^{-2/3}$ (vi) $\log_b \sqrt{xy}$

(vii) $\log_2 5x^2y/q^3$ (viii) $\log_b (x^2\text{-}y^3)$ (ix) $\log_b x^2/xy$

8. Write the given expressions as a single logarithm.

(i) $\log_b x + \log_b y$ (ii) $2 \log_a m - \log_b x + 3\log_b y)$

(iii) $\log_a y + \log_a x$ (iv) $3 \log_a m - 2 \log_b n$

(v) $\frac{1}{3} \log_b x + \frac{1}{4} \log_b y - 1/6 \log_b z$

9. Given: $\log_a 2 = 1.2$, $\log_a 3 = 1.42$, and $\log_a 5 = 2.1$

Evaluate the following logarithms using the information given above:

(i) $\log_a 100$ (ii) $\log_a 15$ (iii) $\log_a 20$

(iv) $\log_a 1/3$ (v) $\log_a 10$ (vi) $\log_a 3/5$

10. Approximate x to three significant figures.

Find the following logarithms:

(i) $\log x = 3.6274$ (ii) $\log x = 1.8965$

(iii) $\log x = 0.9469$ (iv) $\ln x = -5$

(v) $\ln x = -1.6$ (vi) $\log x = -1.6253$

(vii) $\ln x = 0.95$

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